METR 5004: Problem Set for NWP Fall 2014

Reading Assignment:

Read my numerical analysis notes available from the web site(s). You can skip the last section that starts the diffusion equation.

Also good reference: Chapter 2, section 2.5.2-2.5.3, Numerical Methods for Wave Equations in Geophysical Fluid Dynamics, D. Durran.

P1. Using the expression for the upwind scheme, plot as a function of courant number $c_r = \frac{U\Delta t}{\Delta x}$, and wavenumber what the amplitude response is for the upwind scheme.

$$|\lambda| = \left(1 - 2c_r\left(1 - \cos\left(k\Delta x\right)\right)\left(1 - c_r\right)\right)^{1/2}$$

This is a 2D contour plot, so divide each axis into 11 bins, 0-1 for c_r, and 0-π for the wavenumber. Use a contour interval of 0.01. At what courant number is the damping a maximum?

P2. A well-known advection scheme is called the Lax-Wendroff or the Crowley scheme. Its finite difference form in 1-dimension is given as:

$$f_j^{n+1} = f_j^n - \frac{c\Delta t}{2\Delta x}\left(f_{j+1}^n - f_{j-1}^n\right) + \frac{1}{2}\left(\frac{c\Delta t}{\Delta x}\right)^2\left(f_{j+1}^n - 2f_j^n + f_{j-1}^n\right)$$

where "c" is the a constant velocity advection.

a) Plug in a Talyor series expansion for (1) and determine the order of accuracy in space and time for this scheme.
b) Find the amplification matrix and show what conditions are necessary for stability.
c) If the analytical phase speed is given by

$$\theta_a = -kc\Delta t$$

derive an expression for the ratio of the numerical phase speed:

$$\frac{\theta_{LW}}{\theta_a}$$

d) for a 4 Δx wave, c = 0.25 and Δt = Δx = 1.0, calculate the phase speed ratio above.